Estimations of High-Energy Photopion Production at 0°

URI MAOR
Stanford Linear Accelerator Center, Stanford University, Stanford, California*
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Experimental measurements indicate a much higher cross section than one-pion-exchange (OPE) predictions for high-energy charged-pion photoproduction at very small angles. Some possible mechanisms for 0° high-energy pion photoproduction are examined. It is shown that double pion photoproduction via a Reggeized ρ-exchange process may yield a considerable cross section at $\theta_\pi=0°$. The parameters of the ρ trajectory are determined and compared with other calculations. Since the residue of the ρ trajectory is unknown, an experiment in a deuterium bubble chamber is suggested. The results are applied also to higher photon energies.

I. INTRODUCTION

PHOTOPRODUCTION was suggested a few years ago as a useful technique to produce collimated beams of charged particles (especially charged pions), and for use in investigating strong interactions. According to the one-pion-exchange (OPE) calculation of Drell[1] it is expected, for small momentum transfers, that most of the produced high-energy pions ($w_\pi>\frac{1}{2}K$, where w_π is the pion energy, and K the photon energy) will collimate within the forward angular cone with a sharp peaking of the differential cross section at an angle $\theta_\pi\approx m_\pi/w_\pi$. However, this cross section vanishes at $\theta_\pi\rightarrow 0°$ and the angular distribution at small angles is proportional to $\sin^2\theta_\pi/(1-\beta_\pi\cos\theta_\pi)^2$, β_π being the produced π velocity in c units.

Some experimental measurements on a Be target were carried out[2] in order to examine these predictions for charged pions at lab angles of 1° to 11°, and photon energies between 2 and 6 BeV. The experiment verified that most of the pions were indeed produced in the narrow forward cone, but yielded surprisingly high cross sections at very small angles. At $\theta_\pi\approx m_\pi/w_\pi$ the observed cross sections grow about two times bigger than the predicted values, and there is no observed tendency to decrease toward $\theta_\pi\rightarrow 0°$. OPE calculations, including the effects of Fermi motion, show an effect much too small to be compared with the recorded cross sections in the forward direction.

These measurements were performed with counters, leaving considerable uncertainty about the clean separation between the pion and electron yields. Also, the effects of final-state interactions were not included in the analysis of the experimental results. However, even with these reservations, it seems that one cannot ignore the possibility of a considerably large cross section for high-energy photo pion production in the strict forward direction due to processes other than OPE. Moreover, angular-momentum conservation rules out the most promising peripheral candidates in 0° high-energy pion photoproduction. It is thus interesting to examine some possible photoproduction mechanisms that might produce high-energy pions at 0°. Such calculations may serve as an aiding test of the reliability of the reported experiment,[2] and help further investigations of the possibility to produce secondary collimated beams from high-energy electron accelerators.

In this paper the discussion is limited to the framework of peripheral models. In Sec. II we consider pion production in the forward direction as a decay product of a photo vector meson production. Section III is devoted to some other contributions to the double pion photoproduction, and in Sec. IV the results are discussed and applied to very energetic photons (20 BeV).

II. CONTRIBUTIONS FROM ρ PHOTOPRODUCTION

Examination of the main peripheral contributions to the single pion photoproduction (see Fig. 1)

$$\gamma+A\rightarrow\pi+B \tag{1}$$

shows[3–5] that the calculated cross sections at 0° remain either zero, or, in the case of the ρ exchange [Fig. 1(b)], extremely small as compared with the OPE contribution at $\theta_\pi\approx m_\pi/w_\pi$. Since the experimental setup[2] was arranged to detect charged pions of about 1 BeV below the incident photon energy, one should bring into account also the effects of double pion photoproduction.

Let us consider first the case of double pion production via a ρ photoproduction:

$$\gamma+N\rightarrow\rho+N,\quad \rho\rightarrow\pi+\pi. \tag{2}$$

If the ρ is photoproduced by exchange of a spin-0 object, it will retain the incoming photon transverse polarization, leading therefore to a $\sin^2\theta_\pi$ dependence of the differential cross section for the decay product pions. The same was shown to be true in the case of a multiperipheral model.[4] In the case of a ρ-exchange mecha-

* Supported by the U. S. Atomic Energy Commission.
[1] S. D. Drell, Phys. Rev. Letters **5**, 278 (1960).
[2] R. B. Blumenthal, W. L. Faissler, P. M. Joseph, L. J. Lanzerotti, F. M. Pipkin, D. G. Stairs, J. Ballam, H. De Staebler, Jr., and A. Odian, Phys. Rev. Letters **11**, 496 (1963).
[3] A. P. Contogouris, F. Hadjioannou, and T. Kinoshita, Nuovo Cimento **29**, 192 (1963).
[4] S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964).
[5] G. Kramer and P. Stichel, Z. Physik (to be published).

nism leading to a ρ photoproduction [Fig. 2(a)] one will avoid the $\sin^2\theta_\pi$ dependence of the other mechanisms, assuming that the ρ has a finite g factor. Considering the magnetic moment coupling between the ρ and the photon, it was shown by Berman and Drell,[4] that after integrating over one of the produced pions one is left with the following differential cross section:

$$\frac{d^2\sigma}{dw_1 d\Omega_1} \approx \frac{(1+\mu_\rho)^2}{\pi}\left(\frac{g_{\rho NN}^2}{4\pi}\right)\left(\frac{w_1^4}{KM^3}\right) \times\left\{\frac{m_\rho^2}{8w_1^2}+\sin^2\frac{\theta_1}{2}\left(1-\frac{K}{2w_1}\right)\right\}F_\pi(\theta_1,w_1)\frac{\text{mb}}{\text{BeV-sr}}, \quad (3)$$

where $1+\mu_\rho$ is the total magnetic moment of the ρ, $g_{\rho NN}^2/4\pi$ its coupling constant, K is the photon energy, M is the nucleon mass, and F_π is dependent on w_1 and θ_1 and should be calculated numerically for each specific case. The ρ-exchange amplitude does not interfere with the OPE, and the two cross sections are additive.

Due to the unknown coupling and magnetic moment of the charged ρ meson this expression is unnormalized, but if we assume

$$1 \leq (1+\mu_\rho)^2(g_{\rho NN}^2/4\pi) \leq 10\,, \quad (4)$$

the resulting cross section in the forward direction is comparable to the OPE contribution. Even if one accepts assumption (4) it is very difficult to interpret

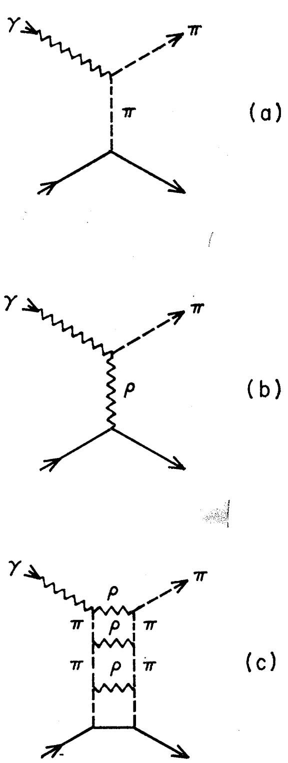

FIG. 1. Some peripheral contributions to single pion photoproduction: (a) one-pion exchange, (b) ρ exchange, (c) multiperipheral model.

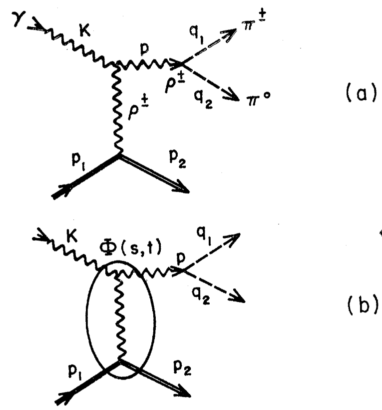

FIG. 2. ρ-exchange mechanism leading to a ρ photoproduction: (a) the exchanged ρ treated as a perturbative pole; (b) the exchanged ρ treated as a Regge pole.

the experimental results by cross section (3): The experiment[2] gave, for $w_\pi \approx K-1$, a very slow increase with K of the differential cross section at 1°, indicating also that any cross section responsible for the photopion production in the strict forward direction should be very peaked in the narrow forward cone to avoid tremendous overestimations at $\theta_\pi > 3°$. These conditions are not satisfied by Eq. (3), which yields a broad angular distribution, and a very strong K dependence of the calculated cross section in the forward cone. Since the K dependence of the cross section at $0° \leq \theta_1 \leq 1°$ is weakly dependent on the momentum transfer t, it is practically impossible to improve the calculations by adding some t-dependent form factors. It is possible to get much better results by treating the exchanged ρ as a Regge pole. Such a calculation, in addition to its connection to photopion production, is interesting in view of the recent arguments in favor and against a similar treatment of the n-p charge exchange at the same energy range.[6–8]

The Reggeized amplitude for reaction (2) [see Fig. 2(b)] is given by

$$M = 2e\epsilon_\mu \Phi(s,t)\frac{g^{\mu\nu}-p^\mu p^\nu/m_\rho^2}{p^2-m_\rho^2-im_\rho\Gamma_\rho}g_{\rho\pi\pi}(q_1-q_2)_\nu\,, \quad (5)$$

ϵ_μ is the photon polarization. $g_{\rho\pi\pi}$ is determined from Γ_ρ—the observed width of the 2π decay mode of the ρ. The time-like photoproduced ρ was not treated as a Regge pole since $p^2 \approx m_\rho^2$.

$$\begin{aligned} s &= (p_1+K)^2 = M^2+2KM\,, \\ t &= (p-K)^2 < 0\,, \\ Z_t &= \frac{s-M^2+t/2}{2(t/4)^{1/2}(t/4-M^2)^{1/2}}\,. \end{aligned} \quad (6)$$

[6] I. J. Muzinich, Phys. Rev. Letters 11, 88 (1963).
[7] R. J. N. Phillips, Phys. Rev. Letters 11, 442 (1963).
[8] D. V. Bugg, Phys. Letters 7, 365 (1963).

In a first approximation for $1/Z_t^2$:

$$\Phi(s,t)=\frac{(2\alpha+1)}{\pi^{1/2}}\beta(t)\frac{1-\exp(-i\pi\alpha)}{\sin\pi\alpha}\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}(2Z_t)_\alpha\,, \quad (7)$$

where $\beta(t)$, $\alpha(t)$ are the residue and position of the ρ trajectory, respectively. Eq. (7) holds for $\alpha\geq-\frac{1}{2}$.[9] The ρ trajectory was approximated by a straight line

$$\alpha(t)=\alpha(0)+t[1-\alpha(0)]/m_\rho^2. \quad (8)$$

The threshold behavior of Φ was removed in the standard manner:

$$\beta(t)=\left(\frac{2(t/4)^{1/2}(t/4-M^2)^{1/2}}{t_0}\right)^\alpha b(t). \quad (9)$$

Inserting these relations into (7) one gets:

$$|\Phi|^2=\frac{b^2}{\pi}\frac{(2\alpha+1)^2}{\cos^2\pi\alpha/2}\left(\frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)}\right)^2\left(\frac{4KM}{t_0}\right)^{2\alpha}. \quad (10)$$

In the p-n charge exchange calculations[6] $\alpha(0)$ was treated as a free parameter, t_0 was assumed to equal the square of the ρ mass, and the t dependence of $b(t)$ was neglected for small $|t|$ values. In the present calculations it is necessary to integrate over a wider range of t. Due to the boundary conditions of (7), contributions from $t<-m_\rho^2[\frac{1}{2}+\alpha(0)]/[1-\alpha(0)]$ were neglected. $b(t)$ was assumed to equal a constant over

$$-m_\rho^2[\tfrac{1}{2}+\alpha(0)]/[1-\alpha(0)]<t<0,$$

and $\alpha(0)$, t_0 were treated as free parameters. The differential cross section in the limit of no recoil is given, then, by:

$$\frac{d^2\sigma}{dw_1d\Omega_1}=\frac{1}{2\pi^2}\frac{e^2}{4\pi}\frac{g_{\rho\pi\pi}{}^2}{4\pi}\frac{d\Omega_2}{4\pi}\frac{|\mathbf{q}_1||\mathbf{q}_2|}{K}|\Phi|^2\Theta\left(t+m_\rho^2\frac{\frac{1}{2}+\alpha(0)}{1-\alpha(0)}\right)$$
$$\times\frac{|\mathbf{q}_1|^2\sin^2\theta_1+|\mathbf{q}_2|^2\sin^2\theta_2-2|\mathbf{q}_1||\mathbf{q}_2|\sin\theta_1\sin\theta_2\cos(\varphi_2-\varphi_1)}{(p^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2}\,, \quad (11)$$

where

$$\Theta(x)=1:\ x\geq1$$
$$=0:\ x<1.$$

Cross section (11) was calculated numerically for several values of K, with $\alpha(0)$ between 0.1 and 0.9 and t_0 ranging from m_π^2 to $2M^2$. The mass and total width of the ρ were taken as 757 and 120 MeV, respectively.[10] The normalization of (11) depends on the assumed value of $b(t)$ in the physical region. Since the pole and Regge approximations coincide at the ρ pole, the value of $b(t)$ is correlated to $(1+\mu_\rho)^2(g_{\rho NN}{}^2/4\pi)$ via the extrapolation of the residue from the physical region to the pole $t=m_\rho^2$. Assuming that $b(t)$ changes very slowly from the physical region to the pole, the calculated cross section was normalized for $2\leq(1+\mu_\rho)^2(g_{\rho NN}{}^2/4\pi)\leq5$. This assumption is supported by dynamical models[11,12] of the ρ trajectory.

Surprisingly, although the experimental measurements were quite rough, one can draw some general conclusions which do not depend upon the fine details of the experimental setup. It appears that if the ρ exchange plays any appreciable role in double pion photoproduction, the only reasonable fit to experimental results is achieved for $t_0\approx m_\rho^2$ and $0.2<\alpha(0)<0.5$. For $t_0\gg m_\rho^2$ the angular distribution (11) is much too broad as compared with the experimental results, unless one takes $\alpha(0)\approx-0.5$ which is unreasonable. Very small values of t_0 (order of m_π^2) strongly increase the peaking of (11) (for $K=4.85$ BeV, $w_1=4.0$ BeV the calculated value at $\theta_1=4°$ is about 15 times bigger than the value at $\theta_1=1°$), in sharp contradiction to the observed values. Remembering that π- and ρ-exchange cross sections are additive, it is seen that low values of $\alpha(0)$ are favorable since the higher values of $\alpha(0)$ yield much too broad an angular distribution, and strong K dependence of the cross sections in the forward cone.

The calculated curves as compared with the measured experimental points are given in Fig. 3, for $\alpha(0)=0.3$ and $t_0=m_\rho^2$. The lower curve is the OPE contribution and the upper one is the sum of the π and ρ exchanges. Both cross sections were integrated over the γ spectrum in the same way it was done in Ref. 2. In the present status of the experimental results, no serious attempt was made to choose the best fit and the graphs are demonstrative. Generally, a certain overestimation of the calculated cross section is observed at higher values of θ_1. It should be remembered, however, that recoil effects of the target will reduce the calculated curves at $\theta_1\geq4°$.[13] The dashed lines give the corrections based on the extreme assumption that all the recoil momenta are being carried out by a single nucleon. In the present

[9] M. Gell-Mann, *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962).
[10] M. Roos, Phys. Letters **8**, 1 (1964).
[11] H. Cheng and D. Sharp, Phys. Rev. **132**, 1854 (1963).
[12] M. Bander and G. L. Shaw, Phys. Rev. **135**, B267 (1964).

[13] M. Thiebaux, SLAC Report No. 21, Stanford Linear Accelerator Center, Stanford University, Stanford, California, November 1963 (unpublished).

work all comparisons were made relative to the OPE in the pole approximation. Reggeization of the OPE amplitude at these energies will cause only a small reduction at very small angles,[3] but the effect at much higher energies or higher θ_1 values is appreciable.[14] As for the very small angles, the separation between the measured pions and electrons was very difficult[2] and reduction of the presently reported experimental cross sections is very probable.

The dependence of (11) on Γ_ρ is given in Fig. 4. As can be seen, the calculated cross section is not sensitive to Γ_ρ changes. This point will be emphasized when comparing our results with some dynamical models of the ρ meson trajectory.

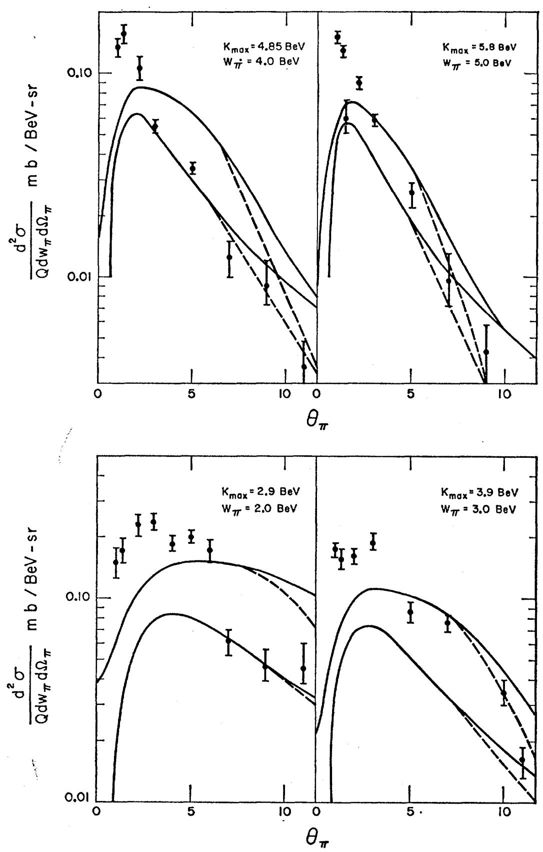

FIG. 3. The calculated cross sections as compared with experiment, θ_π is measured in degrees.

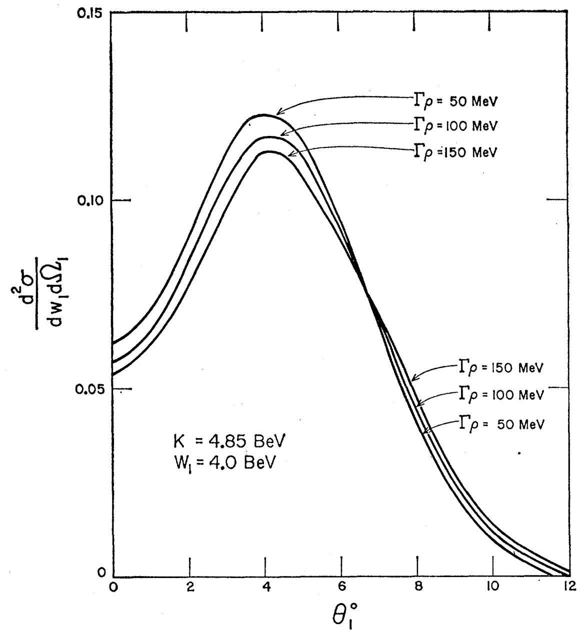

FIG. 4. Γ_ρ dependence of the differential cross section contributed by ρ exchange, θ_1 is measured in degrees, and $d^2\sigma/dw_1 d\Omega_1$ is given in arbitrary units.

III. OTHER CONTRIBUTIONS TO DOUBLE PION PHOTOPRODUCTION

It was seen that the Reggeized ρ-exchange process may yield a considerable cross section for π production in the strict forward direction $\theta_1 = 0°$. It is desirable to examine other possible contributions at 0° in order to apply an experimental verification of the ρ-exchange importance.

Let us consider the general gauge-invariant expresion for the differential cross section.[15] For a real transversely polarized photon the cross section for double pion photoproduction (Fig. 5) will be given by:

$$
\begin{aligned}
d\sigma = \epsilon_\nu \epsilon_\mu \Big\{ & A_1(s,t_1,t_2,t)\left(q_1^\mu - \frac{K\cdot q_1}{K\cdot p_1}p_1^\mu\right)\left(q_1^\nu - \frac{K\cdot q_1}{K\cdot p_1}p_1^\nu\right) \\
& + A_2(s,t_1,t_2,t)\left(q_2^\mu - \frac{K\cdot q_2}{K\cdot p_1}p_1^\mu\right)\left(q_2^\nu - \frac{K\cdot q_2}{K\cdot p_1}p_1^\nu\right) \\
& + A_3(s,t_1,t_2,t)\left[(q_1-q_2)^\mu - \frac{K\cdot(q_1-q_2)}{K\cdot(q_1+q_2)}(q_1+q_2)^\mu\right] \\
& \times\left[(q_1-q_2)^\nu - \frac{K\cdot(q_1-q_2)}{K\cdot(q_1+q_2)}(q_1+q_2)^\nu\right] \\
& + \text{spin flip term}\Big\}, \quad (12)
\end{aligned}
$$

[14] It is also important to notice that no t-dependent form factors were included in this calculation [except for the obvious dependence on $\alpha(t)$]. Such a t dependence will also decrease the calculated cross sections for bigger θ_1 values. In the process of calculations it was also shown that insertion of an exponential t dependence [A. Ahmadzabeh and I. Sakmar, Phys. Rev. Letters **11**, 439 (1963)]:

$$b(t) = b(0)\exp(t/a),$$
$$a \approx 25 m_\pi^2,$$

would not change the general features of (12).

[15] S. M. Berman (to be published).

where

$$\begin{aligned} s&=(K+p_1)^2\,,\\ t_1&=(q_1-K)^2=m_\pi{}^2-2K\cdot q_1\,,\\ t_2&=(q_2-K)^2=m_\pi{}^2-2K\cdot q_2\,,\\ t&=(p_2-p_1)^2=t_1+t_2+2q_1\cdot q_2\,. \end{aligned} \tag{13}$$

Ignoring the spin flip amplitude, and since

$$\epsilon_\mu[q_1{}^\mu-(K\cdot q_1/K\cdot p_1)p_1{}^\mu]=|\mathbf{q}_1|\ \sin\theta_1\,, \tag{14}$$

it is suggestive that the cross section at $\theta_1=0°$ originates from the second and third terms of (12),[16] i.e., we have to consider either processes in which the photon is absorbed by the π_2 current and the high-energy π_1 is produced somewhere else, or processes where the photon is coupled to π_1 and π_2 (as was the case in the ρ-exchange calculations).

The possibility of a high-energy π to be emitted from a nucleon or an isobar (Fig. 6) is very unlikely due to kinematical constraints. More generally, if we consider a diffraction mechanism at the lower vertex (Fig. 7),

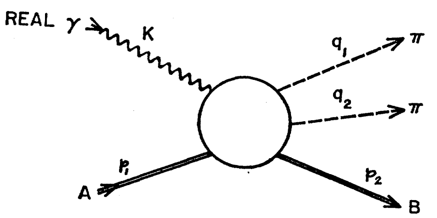

FIG. 5. Double pion photoproduction.

we obtain:

$$\frac{d^2\sigma}{dw_1d\Omega_1}=\frac{1}{2\pi}\frac{e^2}{4\pi}\frac{\sin^2\theta_2}{(1-\beta_2\cos\theta_2)^2}g(t)\frac{d\Omega_2}{4\pi}\frac{w_1{}^3w_2}{K^3}\frac{\sigma_{\text{tot}}(s_1)}{(4\pi)^2}\,, \tag{15}$$

where, following the notation of Drell and Hiida,[17]

$$\begin{aligned} g(t)&=(1-t/\alpha m_\pi{}^2)^{-2},\quad \alpha\approx 10\\ s_1&=(p_2+q_1)^2\,,\\ \beta_2&=|\mathbf{q}_2|/w_2\,. \end{aligned} \tag{16}$$

$\sigma_{\text{tot}}(s_1)$ is the total cross section for the pion-nucleon system and is approximately 30 mb for $s_1\geq 4$ BeV^2. The contribution of (15) in the strict forward direction $\theta_1=0°$ is negligible as

$$R=\frac{(d^2\sigma/dw_1d\Omega_1)^{\rho\ \text{ex.}}{}_{\theta_1=0°}}{(d^2\sigma/dw_1d\Omega_1)^{\text{d.sc.}}{}_{\theta_1=0°}}\gg 10\,. \tag{17}$$

Finally, we consider electromagnetic pion-pair photoproduction. As most of the pair production at this

[16] Generally, it is possible that the form factor A_1 has a $\sin^{-1}\theta_1$ singularity, contributing, therefore, to the cross section at $\theta_1=0°$ This possibility is not discussed in this work.

[17] S. D. Drell and K. Hiida, Phys. Rev. Letters **7**, 199 (1961).

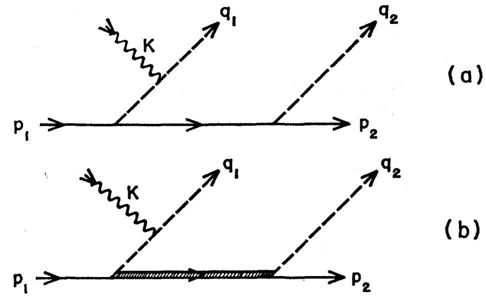

FIG. 6. High-energy photopion production from a nucleon (a) or an isobar (b).

energy range takes place with $|t|<\frac{1}{2}m_\pi{}^2$, we can apply the Pauli-Weisskopf cross section[18] (see Fig. 8):

$$d^2\sigma=Z^2\left(\frac{e^2}{4\pi}\right)^3\frac{[(m_\pi/w_1)^4+\sin^4\theta_1]}{(1-\beta_1\cos\theta_1)^4}\frac{K-w_1}{w_1K^3}\times(\ln B-\tfrac{1}{2})\frac{d\Omega_1}{4\pi}dw_1\,, \tag{18}$$

where

$\beta_1=|\mathbf{q}_1|/w_1$

$B=\dfrac{w_1}{m_\pi}$: for a point charge (a)

$=\dfrac{12w_1(K-w_1)}{m_\pi KZ^{1/3}}$: for a uniform charge distribution over a sphere nucleus. (b)

This cross section is extremely peaked at $\theta_1=0°$, and may be neglected for higher θ_1 values. For $K=6$ BeV the differential cross section at 0° is about 3×10^{-2} mb/BeV-sr under assumption (a), and about 7×10^{-3} mb/BeV-sr under assumption (b). The correct value for a Be target is between these values and may be calculated by inserting the known electromagnetic form factors of Be.

Contributions to electromagnetic pion pair production with a ρ in the intermediate state (Fig. 9) can be

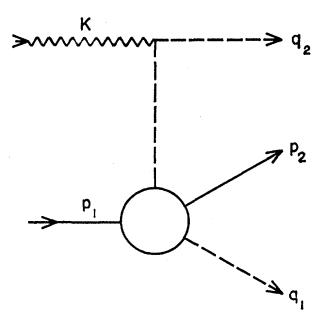

FIG. 7. High-energy photopion production via a diffraction scattering process.

[18] W. Pauli, Rev. Mod. Phys. **13**, 203 (1941). This paper contains an extra factor of 2 that should be omitted.

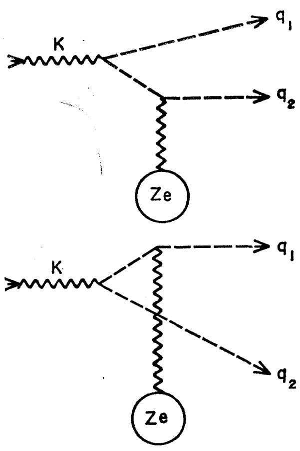

FIG. 8. Electromagnetic pion pair production.

neglected since $g_{\rho\pi\gamma}^2/4\pi$ is of the order of $e^2/4\pi$ or smaller[4] and $m_\rho^2 \gg m_\pi^2$.

To summarize the results: we see that the main contribution to photoproduction of charged pions in the strict forward direction probably comes from a ρ photoproduced via a ρ-exchange mechanism. A much smaller contribution is due to electromagnetic pion-pair production. The other contributions that have been discussed gave either a zero or a negligible cross section at $\theta_1 = 0°$.

IV. DISCUSSION

The analysis presented in this paper suggests that the production of a high-energy charged pion at very small angles in the BeV region is mainly contributed by a photo-ρ production via a Reggeized ρ-exchange process. Keeping in mind the order of magnitude of the electromagnetic cross section (18), any cross section bigger than 10^{-2} mb/BeV-sr at $\theta_1 = 0°$ would support this conclusion. However, this conclusion depends very crucially on two unknown factors: the magnetic moment and the coupling constant of the ρ meson, and the extrapolation of $b(t)$ from the physical region to the pole $t = m_\rho^2$.

A very clean experimental test of this result may be obtained in a deuterium bubble-chamber experiment. Since only $\rho^\pm$ exchanges may contribute to a ρ photoproduction, the forward direction charged π is accompanied by a neutral π, the angular distribution of which can be deduced from (11). The assumed total cross section for the reaction

$$\gamma + D \to \rho^- + p + (p) \qquad \rho^- \to \pi^- + \pi^0, \tag{19}$$

in the very narrow forward cone, is of the order of 0.01 mb. Reaction (19) can be easily detected with a good energy resolution deduced from the recoiled proton. Such an experiment, even if successful, is not sufficient to determine $g_{\rho NN}^2/4\pi$ and μ_ρ simultaneously. Since $g_{\rho NN}^2/4\pi$ can be determined from other independent experiments, reaction (19) may serve as a clue in estimating the magnetic moment of the ρ.

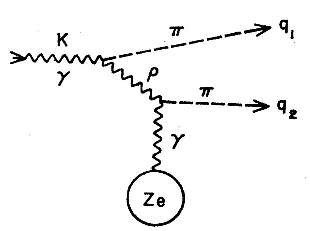

FIG. 9. Electromagnetic pion pair production with an intermediate ρ.

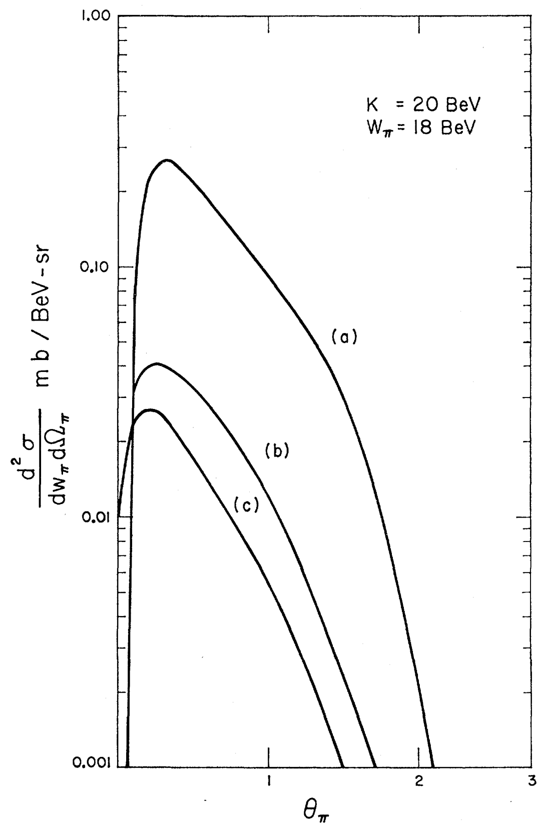

FIG. 10. Cross sections for high-energy photopion production at small angles: (a) OPE in the pole approximation, (b) Reggeized OPE, (c) Reggeized ρ exchange, θ_π is measured in degrees.

In Sec. II it was shown that the experimental results do not agree with the pole approximation for the ρ-exchange process, whereas the Reggeized expression (7) gives a much better fit. The interesting point is that if the ρ-exchange process plays any appreciable role in double pion photoproduction, the parameters of the ρ-exchange amplitude can be determined from the very rough behavior of the differential cross section at small angles. The values $0.2 < \alpha_\rho(0) < 0.5$, $t_0 \approx m_\rho^2$, obtained in our analysis, are in very good agreement with phenomenological analyses of p-n charge exchange[6] and π-p

scattering.[19] Dynamical calculations[11,12] of the π-π scattering amplitude consistently yield a different result: $\alpha_\rho(0) \gtrsim 0.9$. It should be noted, however, that the dynamical calculations of $\alpha_\rho(0)$ are very sensitive to Γ_ρ and were performed for the π-π elastic channel only. The analysis given in this paper is not sensitive to Γ_ρ (see Fig. 4); neither are the p-n charge exchange and π-p scattering analyses.

Let us now examine the consequences of the present analysis for higher photon energies (15–25 BeV). Due to the choice $0.2<\alpha_\rho(0)<0.5$, the ρ-exchange contribution to double pion photoproduction drops slowly with increasing energy. Hence, the differential cross section to photoproduce a high-energy ($K-w_\pi \approx 2$ BeV) charged pion at $\theta_\pi=0°$ will be about 10^{-2} mb/BeV-sr for $K=20$ BeV. This cross section is comparable to the electromagnetic differential cross section (18). Since the energy dependence of photoproduction cross sections is not very well known, it is very speculative to compare these numbers with the Drell cross section. An un-Reggeized π-exchange process contributes a cross section of about 0.3 mb/BeV-sr at $\theta_\pi \approx m_\pi/w_\pi$. Reggeization of this contribution reduces this number by one order of magnitude.[3] Comparison of these cross sections is given in Fig. 10 for $K=20$ BeV, $w_\pi=18$ BeV. As can be seen, detection of the ρ-exchange cross section at $\theta_\pi=0°$ is more complicated for higher energies since the angular resolution needed is one-tenth of a degree, as compared with half a degree at $K=5$ BeV.

The considerations given in this paper may be applied to K photoproduction if the correspondence $\pi \to K$, $\rho \to K^*$ is made.

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K-3π Decay and $K_2{}^0$–$K_1{}^0$ Mass Difference on Current-Current Picture*

Riazuddin and A. H. Zimerman†
Department of Physics and Astronomy, University of Rochester, Rochester, New York
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Some consequences of the current-current picture are considered in connection with K-3π decay and the $K_2{}^0-K_1{}^0$ mass difference. It is shown that this picture in boson pole approximation can explain both the decay rates and spectra for the 3π decay modes of K. The $K_2{}^0-K_1{}^0$ mass difference is considered in terms of a boson pole approximation and a two-pion intermediate state. The relevant weak-coupling constants are estimated on the current-current picture. It is shown that the sign of the mass difference ($\delta m=\delta m_{K_2{}^0}-\delta m_{K_1{}^0}$) is positive, which is in agreement with recent experimental indications. The magnitude is somewhat small but it is not excluded by the present experimental situation.

In this paper we wish to discuss K-3π decay and the $K_2{}^0-K_1{}^0$ mass difference on the current-current picture. In an earlier paper,[1] we used this picture in a simple approximation to explain the experimental decay rate of $K_1{}^0 \to \pi^+\pi^-$ and we also showed that the s-wave amplitudes of hyperon nonleptonic decays can be rather well understood on this picture. We now wish to extend similar considerations to K-3π decay and the $K_2{}^0-K_1{}^0$ mass difference. We show that the current-current picture seems to explain these processes as well.

K-3π DECAY

We first consider $K^+ \to \pi^+\pi^+\pi^-$ decay and write its matrix element as

$$M=M_S+M_V,$$

where M_S and M_V denote the s-wave and p-wave parts of the decay amplitude, respectively. We now assume that M_S on the basis of the current-current picture is described by

$$M_S=\frac{G}{\sqrt{2}}[\langle 0|j_\mu{}^A|\pi^+\rangle\langle\pi^+\pi^-|g_\mu{}^A|K^+\rangle + \langle 0|g_\mu{}^A|K^+\rangle\langle\pi^+\pi^-|j_\mu{}^A|\pi^+\rangle], \quad (1)$$

where G is the universal Fermi constant and j^A and g^A are the strangeness-conserving and strangeness-changing axial vector currents, respectively. Denoting the four momenta of K^+, π^+, π^+, and π^- by K, k_1, k_2, and k_3, respectively, we can write (1) as

$$M_S=\frac{G}{\sqrt{2}}[f_\pi q_\mu\langle\pi^+\pi^-|g_\mu{}^A|K^+\rangle+f_K q_\mu'\langle\pi^+\pi^-|j_\mu{}^A|\pi^+\rangle], \quad (2)$$

where $q=K-(k_1+k_3)=k_2$ and $q'=(-k_2)-(k_1+k_3) = -K$ are the relevant momentum transfers and f_π and

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† On leave of absence from Instituto de Física Teorica, São Paulo, Brazil.
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